

NEW METHODS OF MEASURING DIMENSIONS AND  
EVALUATING A SHAPE WITH A FREE-CURVE SURFACE

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ABSTRACT

This paper deals with a proposition and a development of a PN-probe to directly detect the contact point between the PN-probe itself and an object to be measured, and a construction of a system of measuring a positional vector and an unit normal vector at a point of measurement by combining the PN-probe and a three coordinate measuring machine systematically. Also, methods of measuring dimensions and evaluating a shape with a free-curved surface by using the constructed system of measurement are proposed. In the proposed methods, a distance between parallel planes, a radius of a cylinder and a radius of a sphere, can be obtained by measuring positional vectors and normal vectors of two arbitrary points on each object, and an interpolated shape of a patch forming a free-curved surface can be evaluated on the basis of measured values of positional vectors and unit normal vectors of sixteen arbitrary points on a patch.

INTRODUCTION

In measurement of a physical model by using a 3-coordinate measuring machine (CMM), since data detected by a CMM are only coordinates (x, y, z) of the center of a probe, it is important to detect the position of a contact point between a probe and an object to be measured. Especially, detecting the contact position is a very difficult problem to be solved when an object to be measured has a free-curved surface such as a sculpture [1],[2].

Therefore, in order to solve the problem, a probe to directly detect the location of a contact point between a probe and the object to be measured is proposed and developed in this research. The probe is called PN-probe. The detection of the contact point is based on the principle of a spherical potentiometer, so that the mechanism of the PN-probe is very simple. Also, a system of measuring a positional vector and a unit normal vector of a point to be measured is constructed by combining the developed PN-probe with a CNC-CMM systematically.

As applied measurement based on detection of a positional vector and a normal vector by using the constructed system, measuring methods of dimensions and an evaluating method of a shape with a free-curved surface are proposed. In the measuring methods of dimensions, the distance between parallel planes and radii of a cylinder and a sphere can be determined by detecting positional vectors and unit normal vectors of two arbitrary points on each object. In the evaluating method of a shape with a free-curved surface, an interpolated shape of a patch forming a free-curved surface of a physical model can be evaluated in comparison with a computer model by detecting positional vectors and unit normal vectors of sixteen arbitrary points on the patch.

### DEVELOPMENT OF THE PN-PROBE

#### Basic construction and the principle of detection

Figure 1 shows a basic construction of the proposed PN-probe to directly detect a contact point between the PN-probe itself and an object to be measured. The construction is very simple; that is, a resistant thin film is formed on a surface of an insulating ceramic ball and four electrodes are symmetrically allocated on upper parts of the resistant film.

The detection of the contact point is based on the principle of a spherical potentiometer. As shown in figure 2, the four electrodes formed on the resistant film are connected with a constant current source, and also the object to be measured is connected with the source. When the PN-probe is in contact with the object to be measured, a constant current circuit is formed through the resistant film formed on the PN-probe, the

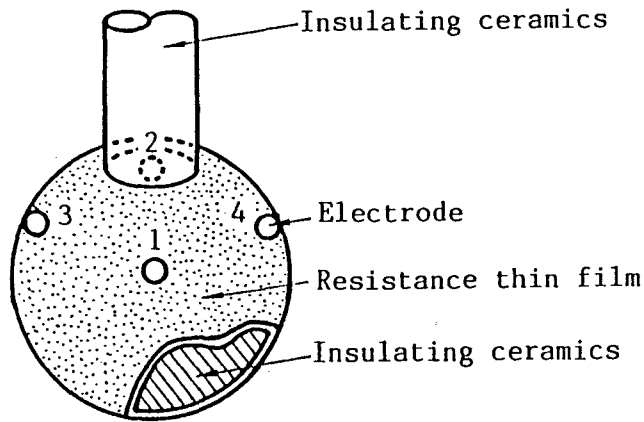


Figure 1. Basic construction of the PN-probe

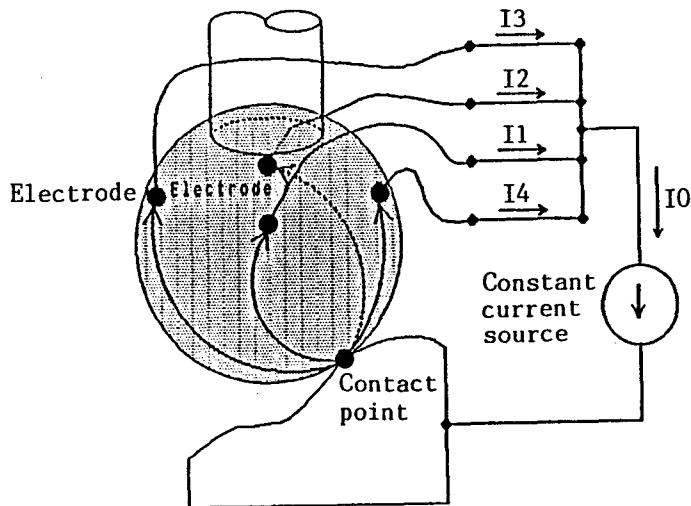


Figure 2. Principle of a spherical potentiometer

constant current source and the object to be measured. In this case, the current in the circuit is distributed in inverse proportion to the resistance values between a contact point and each electrode. Therefore, the location of the contact point can be determined by detecting the

current values flowing into each electrode. The detailed process of the determination is described in reference [3], [4].

#### Development of the PN-probe

Figure 3 shows the developed PN-probe to detect the contact point based on the principle of a spherical potentiometer. The spherical thin film of resistance composed of TiN was deposited on a quartz glass ball with diameter of 10 mm by sputtering equipment. The four electrodes of Au film allocated on the upper parts of the TiN film is also formed by sputtering deposition. The conditions of deposition of the resistant film are shown in table 1, and the characteristics of the deposited resistant film are shown in table 2.

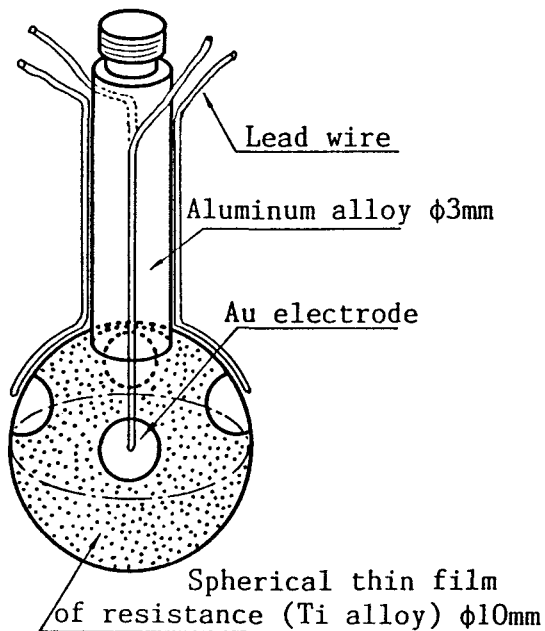


Figure 3. Developed PN-probe

Table 1. Conditions of forming a resistant thin film

Sputtering equipment	RF-Magnetron sputtering
Target	Ti (4N)
Sputtering gas	Ar (4N) $1 \times 10^{-3}$ Toor
Reactive gas	N (4N) $3 \times 10^{-4}$ Toor
Substrate	Quartz glass ball (10mm)
Forming temperature	150 °C
Sputtering electric power	0.8 kW
Sputtering time	3.5 h

Table 2. Characteristics of a resistant thin film

Composition	TiN
Thickness	1.5 $\mu\text{m}$
Temperature coefficient of resistivity (TCR)	650 ppm/°C
Surface roughness	Rmax=0.17 $\mu\text{m}$

### CONSTRUCTION OF THE SYSTEM OF MEASUREMENT

#### A method of measuring a positional vector and a unit normal vector

Figure 4 shows the relationship between the position of the probe center  $P_c$  and the position of the point  $M_p$  to be measured. The point  $M_p$  to be measured is a contact point between the probe and the object to be measured. The positional vector  $V_p$  of the probe center  $P_c$  is detected by a function of a CMM. Since the shape of the PN-probe is a sphere, it means determination of a unit normal vector  $V_n$  of the point  $M_p$  to be measured that the location of the point  $M_p$  to be measured is detected by the developed PN-probe. Therefore, The positional vector  $V_o$  of the point  $M_p$  to be measured is determined by the equation (1) based on the positional vector  $V_p$  and the unit normal vector  $V_n$  which are obtained as measured values.

$$V_o = V_p - r \cdot V_n \quad (1)$$

#### The system of measuring a positional vector and a unit normal vector

The system of measuring a positional vector and a unit normal vector is constructed by combining the developed PN-probe with a CMM systematically; that is, the PN-probe is installed with a touch trigger probe head

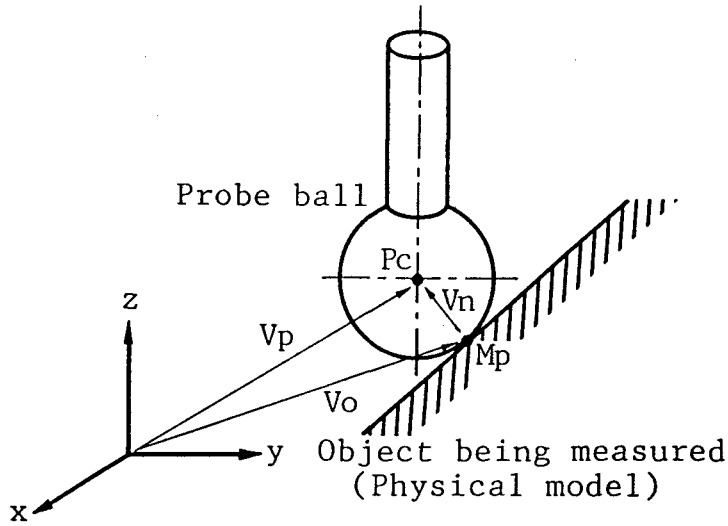


Figure 4. Relationship between a probe center  $P_c$  and a point to be measured

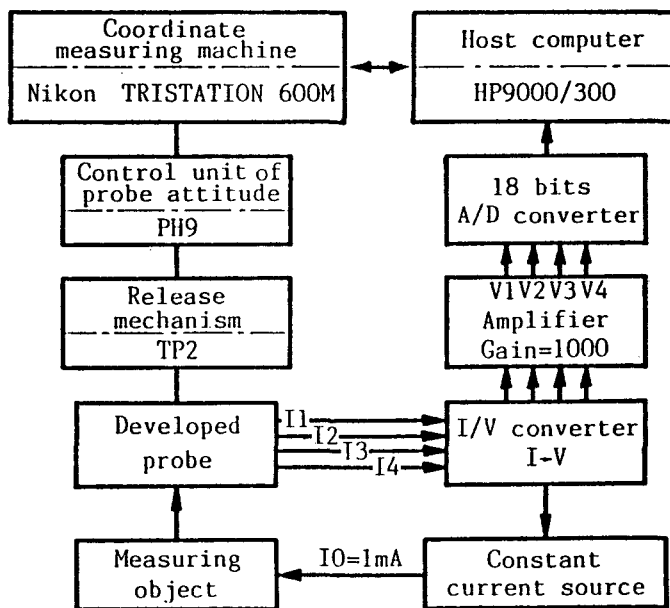


Figure 5. Configuration of the constructed system

(RENISHAW/TP2) and a motorized probe head (RENISHAW/PH9) in a CNC-CMM (NIKKON/TRISTATION600M), and is used as a stylus of the CMM. Figure 5 shows the configuration of the constructed general system. Lead wires from the four electrodes formed on the PN-probe are connected with a negative pole

of a constant current source of which the current is set at 1 mA, and also a lead wire from the object to be measured is connected with a positive pole from the source. The values of the electric currents flowing into each electrode from a contact point are fed into a host computer (HP9000/300) to control the CNC-CMM through an analog-to-digital converter with 18 bits as values of voltage which are obtained by converting the values of the current and amplifying them 1000 times.

The unit normal vector  $V_n$  of the point to be measured is determined according to the interpolation method described in reference [3] on the basis of measured values of voltage as values of electric current flowing into the electrodes. And the positional vector  $V_p$  of a probe center is detected by the CNC-CMM. Therefore the positional vector  $V_o$  of the point to be measured can be determined from the equation (1).

#### Measuring accuracy of a unit normal vector of the system

As shown in figure 6, the spherical coordinate  $(\theta, \phi)$  with the bases of the probe center and the electrode 1 is defined in order to indicate a unit normal vector.

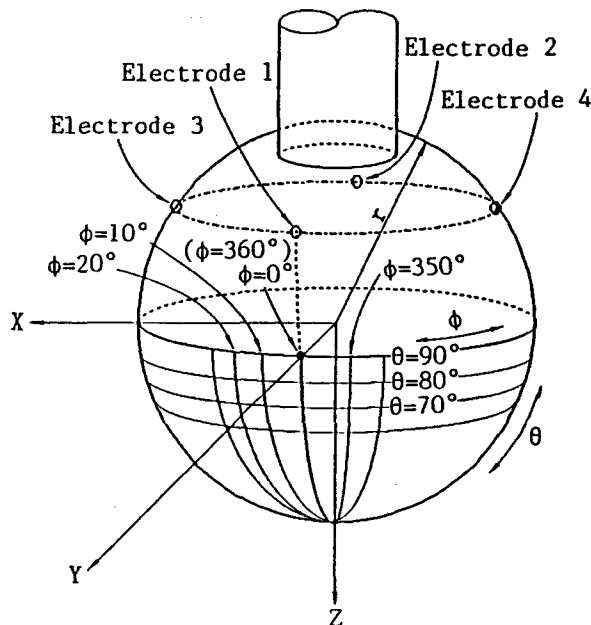


Figure 6. Spherical coordinate of indicating an unit normal vector

An experiment to confirm the measuring accuracy of the constructed system on a unit normal vector was performed. Measurement errors are obtained by measuring a standard surface with a unit normal vector of 60 degrees in the  $\theta$ -angle and 10-degree intervals from 180 degrees to 360 degrees in  $\phi$ -direction. Figures 7 (a) and (b) show the results of measurement errors obtained from the experiment. From these figures, it is clarified that the system has the measuring accuracy under 0.1 degree in the  $\theta$ -angle and  $\phi$ -angle.

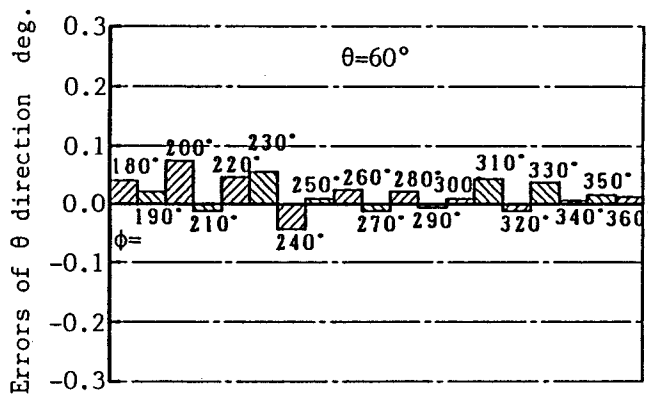
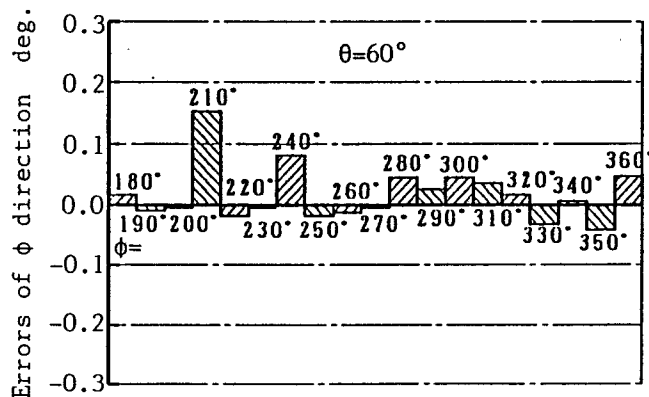
(a) Errors on  $\theta$ -direction(b) Errors on  $\phi$ -direction

Figure 7. Results of measurement errors



NEW METHODS OF MEASUREMENT OF DIMENSION

A method of measuring a distance between parallel planes

A method of measuring the distance between parallel planes  $S1$  and  $S2$  shown in figure 8 based on detecting positional vectors and unit normal vectors is proposed. A positional vector  $p1$  and a unit normal vector  $n1$  of an arbitrary point  $Q1$  on a plane  $S1$  and a positional vector  $p2$  and a unit normal vector  $n2$  of an arbitrary point  $Q2$  on a plane  $S2$  are obtained as measured values. In figure 8,  $\theta1$  represents the angle between a straight line  $L1$ , which is passing the point  $Q1$  and parallel to the vector  $n1$ , and the vector  $(p2-p1)$ , and also  $\theta2$  represents an angle between a straight line  $L2$ , which is passing the point  $Q2$  and parallel to the vector  $n2$ , and the vector  $(p2-p1)$ . Therefore, the next equations (2) are obtained by the measured vectors  $p1$ ,  $p2$ ,  $n1$  and  $n2$ .

$$\left. \begin{aligned} n1 \cdot (p1 - p2) &= |n1| |p1 - p2| \cos(\theta1) \\ n2 \cdot (p2 - p1) &= |n2| |p2 - p1| \cos(\theta2) \end{aligned} \right\} \quad (2)$$

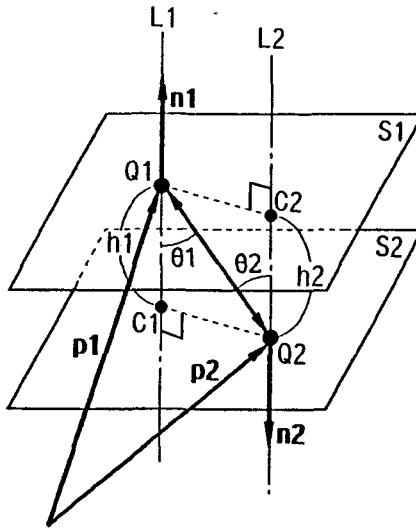


Figure 8. Distance between parallel planes

The distance  $h1$  between the point  $Q1$  and the point  $C1$ , which is a point of intersection of the line  $L1$  and the plane  $S2$ , and the distance  $h2$  between the point  $Q2$  and the point  $C2$ , which is a point of intersection of

the line L2 and the plane S2, are obtained from the equations (3).

$$\left. \begin{aligned} h_1 &= |p_1 - p_2| \cos(\theta_1) \\ h_2 &= |p_2 - p_1| \cos(\theta_2) \end{aligned} \right\} \quad (3)$$

In this case, the distances  $h_1$  and  $h_2$  represent the distance between the planes S1 and S2. Since the vectors  $n_1$  and  $n_2$  have a magnitude of one, the distances  $h_1$  and  $h_2$  are derived as the next equations (4) from the equations (2) and (3) by using the only measured vectors  $p_1$ ,  $n_1$ ,  $p_2$  and  $n_2$ .

$$\left. \begin{aligned} h_1 &= |n_1 \cdot (p_1 - p_2)| \\ h_2 &= |n_2 \cdot (p_2 - p_1)| \end{aligned} \right\} \quad (4)$$

When the distance  $h_1$  is different from the distance  $h_2$  because of errors of machining and measurement, the average is determined as the distance between the parallel planes.

#### A method of measuring a radius of a cylinder

A method of measuring a radius of a cylinder by detecting positional vectors and unit normal vectors of two arbitrary points on a cylinder is proposed. As shown in figure 9, a positional vector  $p_1$  and a unit normal vector  $n_1$  of an arbitrary point Q1 and a positional vector  $p_2$  and a unit normal vector  $n_2$  of an arbitrary point Q2 are obtained as measured values. In figure 9, the point Q0 is an intersection of the center axis of a cylinder and a straight line which is passing the point Q1 and parallel to the vector  $n_1$ , and also the point Q3 is an intersection of the center axis and a straight line which is passing the point Q2 and parallel to the vector  $n_2$ . The positional vector  $p_0$  of the point Q0 and the positional vector  $p_3$  of the point Q3 are obtained from the equations (5), (6) by using the vectors  $p_1$ ,  $n_1$ ,  $p_2$  and  $n_2$  and a radius  $r$  of the cylinder.

$$p_0 = p_1 - r \cdot n_1 \quad (5)$$

$$p_3 = p_2 - r \cdot n_2 \quad (6)$$

A unit vector  $n_0$  parallel to a center axis of the cylinder is obtained from the equation (7).

$$n_0 = n_1 \times n_2 \quad (7)$$

The distance  $h$  between the points Q0 and Q3 is expressed by the equation (8).

$$h = n_0 \cdot (p_1 - p_2) \quad (8)$$

The positional vector  $p_3$  is also expressed by the next equation (9).

$$p_3 = p_0 + h \cdot n_0 \quad (9)$$

The next equation (10) is derived from the equations (6) and (9).

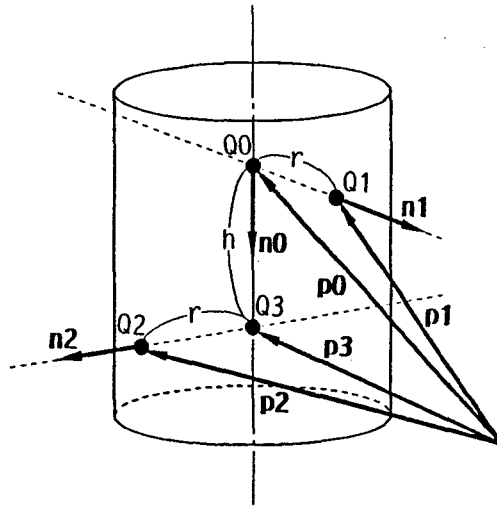


Figure 9. Radius of a cylinder

$$p_0 = p_2 - r \cdot n_2 - h \cdot n_0 \quad (10)$$

And the next equation (11) is derived from the equations (5) and (10).

$$r = \frac{|(p_2 - p_1) - h \cdot n_0|}{|n_2 - n_1|} \quad (11)$$

where, the vector  $n_1$  is not equal to the vector  $n_2$ .

Therefore, a radius  $r$  of a cylinder can be expressed as the equation (12) by using the only measured vectors  $p_1$ ,  $n_1$ ,  $p_2$  and  $n_2$  from the equations (7), (8) and (11).

$$r = \frac{|p_2 - p_1 - \{(n_1 \times n_2) \cdot (p_1 - p_2)\} (n_1 \times n_2)|}{|n_2 - n_1|} \quad (12)$$

#### A method of measuring a radius of a sphere

A method of measuring a radius of a sphere by detecting positional vectors and unit normal vectors of two arbitrary points on a sphere is proposed. As shown in figure 10, when a positional vector  $p_1$  and a unit normal vector  $n_1$  of an arbitrary point  $Q_1$  on a sphere are obtained as measured values, the positional vector  $p_c$  of the center  $O_c$  of a sphere is given by the next equation (13).

$$p_c = p_1 - r \cdot n_1 \quad (13)$$

where,  $r$  is a radius of a sphere. Also, when a positional vector  $p_2$  and a unit normal vector  $n_2$  of an arbitrary point  $Q_2$  on a sphere are obtained as measured values, the positional vector  $p_c$  of the center  $O_c$  is given by the next equation (14).

$$p_c = p_2 - r \cdot n_2 \quad (14)$$

Therefore, a radius  $r$  of a sphere is derived as the equation (15) by using the only measured vectors  $p_1, n_1, p_2$  and  $n_2$  from the equations (13) and (14).

$$r = \frac{|p_2 - p_1|}{|n_2 - n_1|} \quad (15)$$

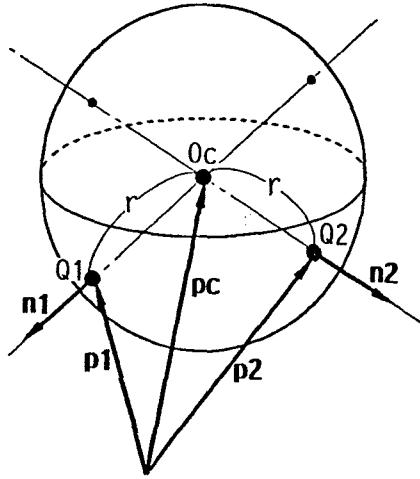


Figure 10. Radius of a sphere

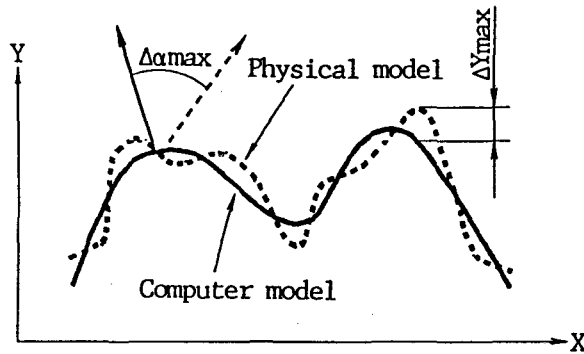
#### A METHOD OF EVALUATING A FREE-CURVED SURFACE

##### Criterion of evaluation of a free-curved surface

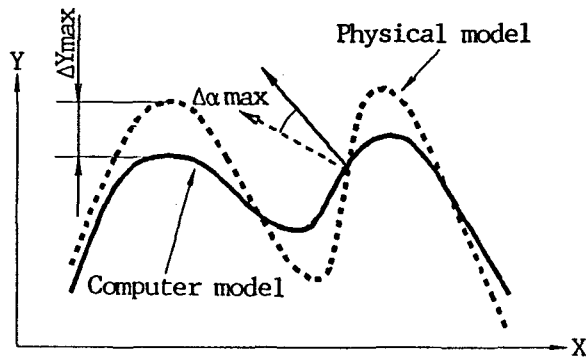
Figures 11 (a) and (b) show two types of physical models; that is, one shown in figure 11 (a) has a large error on a positional vector and a small error on a unit normal vector and the other shown in figure 11 (b) has a small error on a positional vector and a large error on a unit normal vector in comparison with its computer model. In this case, the results of

the evaluations of the two physical models must be different according to the purpose of use. If appearances of the used physical model are more important than positions of the surface, the physical model shown in figure 11 (b) is valued higher than that of figure 11 (a). Furthermore, if positions of the surface of the used physical model are more important than appearances, the physical model shown in figure 11 (a) is valued higher than that of figure 11 (b).

Therefore, a machined physical model must be evaluated according to the two criteria: that is, a positional vector and a unit normal vector. The rate of a positional vector and a unit normal vector in the evaluation is determined on the basis of the purpose of use of the model.



(a) High evaluation model on a positional vector



(b) High evaluation model on a unit normal vector

Figure 11. Criterion of evaluation of a free-curved surface

### A method of evaluating the physical model with a free-curved surface

The constructed measuring system based on the developed PN-probe and a CNC-CMM is able to detect a positional vector and a unit normal vector of an arbitrary point on a surface of a physical model. In this research, a method of evaluating an interpolated shape of a patch forming a free-curved surface based on measuring positional vectors and unit normal vectors of sixteen points on the patch is proposed.

Figure 12 shows a flow chart of the method of evaluating an interpolated shape of a patch. Interpolating equations of a positional vector  $C_p$  and a unit normal vector  $C_n$  of a patch forming a computer model are constructed by using parameters  $u$  and  $v$ ; that is, the positional vector  $C_p$  is given as  $f_x(u,v)$ ,  $f_y(u,v)$  and  $f_z(u,v)$  and the normal vector  $C_n$  is given as  $g_x(u,v)$ ,  $g_y(u,v)$  and  $g_z(u,v)$ . In this case, the relevant patch is considered as the Coons patch or the Bezier patch. On the other hand, positional vectors  $M_{pi}$  ( $q_{xi}$ ,  $q_{yi}$ ,  $q_{zi}$ ) and unit normal vectors  $M_{ni}$  ( $t_{xi}$ ,  $t_{yi}$ ,  $t_{zi}$ ) of sixteen arbitrary points on a surface corresponding to the patch to be evaluated ( $i$  is the number of the points to be measured: that is, 16) are obtained as measured values by the measuring system.

In order to find out parameters  $u_i$  and  $v_i$  corresponding to the sixteen point to be measured, the original evaluation-equations (16), (17), (18) are introduced.

$$E_{f1} = (f_x(u,v) - q_{xi})^2 + (f_y(u,v) - q_{yi})^2 + (f_z(u,v) - q_{zi})^2 \quad (16)$$

$$E_{f2} = (g_x(u,v) - t_{xi})^2 + (g_y(u,v) - t_{yi})^2 + (g_z(u,v) - t_{zi})^2 \quad (17)$$

$$E_{fn} = w_1 \cdot E_{f1} + w_2 \cdot E_{f2} \quad (18)$$

The  $E_{f1}$  in the equation (16) is the evaluated value on a positional vector, and the  $E_{f2}$  in the equation (17) is the evaluated value on a unit normal vector. The equation (18) is derived to evaluate a patch; that is, the  $E_{fn}$  which is obtained as the sum of the  $E_{f1}$  multiplied by  $w_1$  and the  $E_{f2}$  multiplied by  $w_2$  is the evaluated value on a positional vector and a unit normal vector. The  $w_1$  and  $w_2$  are weight coefficients on a positional vector and a unit normal vector respectively. The sum of  $w_1$  and  $w_2$  is 1, and their values must be determined according to the purpose of use of the model. The parameters  $u_i$  and  $v_i$  corresponding to the points to be measured are estimated by finding out  $u$  and  $v$  which minimize the value of  $E_{fn}$  in the equation (18).

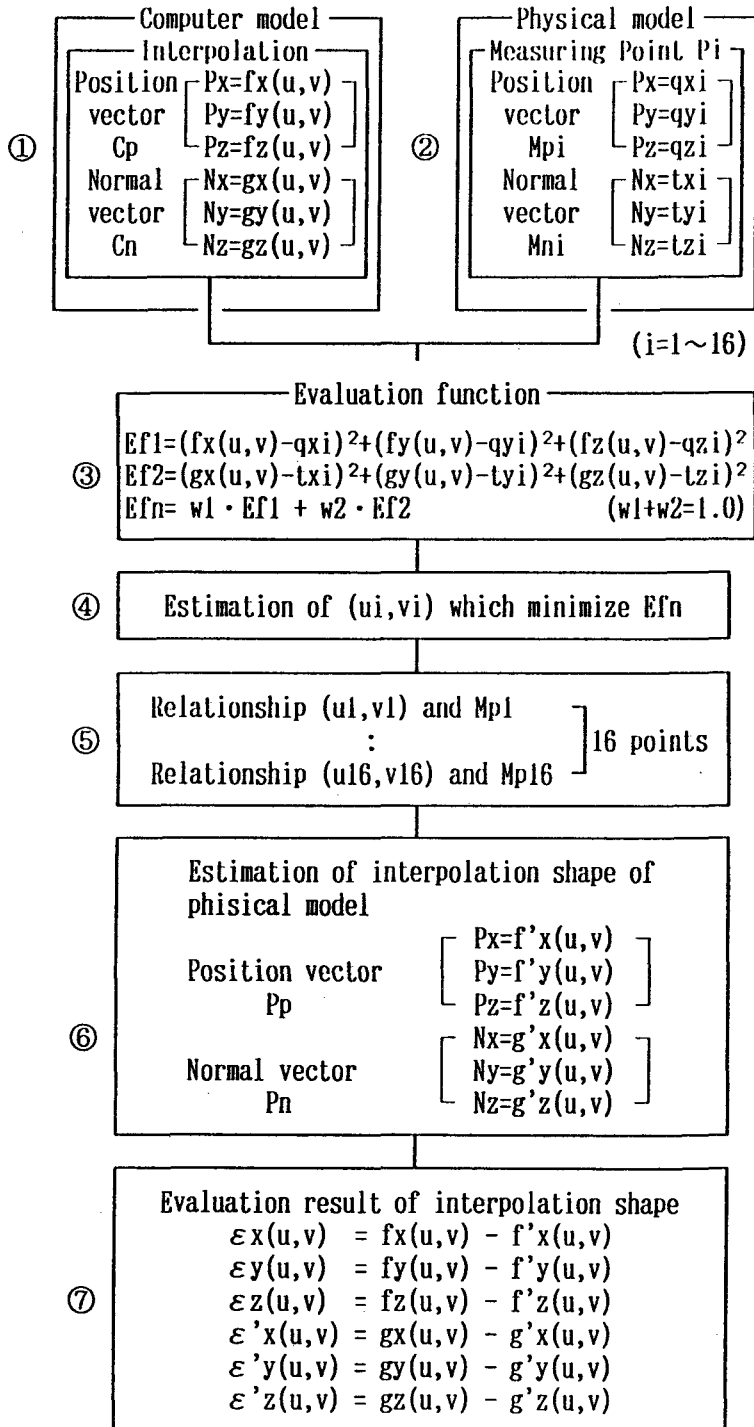


Figure 12. Flow chart of the method of evaluating a patch

The parameters  $u_i$  and  $v_i$  corresponding to sixteen measured positional vector  $M_{pi}$  are obtained as the results of the above procedure. These sixteen relationships between measured positional vectors  $M_{pi}$  and parameters  $u_i, v_i$  are able to derive the interpolating equations of the measured patch; that is,  $f'_x(u,v), f'_y(u,v)$  and  $f'_z(u,v)$  are obtained as the interpolating equations of the positional vector  $P_p$  of the patch being measured, and  $g'_x(u,v), g'_y(u,v)$  and  $g'_z(u,v)$  are obtained as the interpolating equations of the unit normal vector  $P_n$  of the patch being measured.

Therefore, the evaluation of a patch forming a free-curved surface based on a positional vector and a unit normal vector can be performed by comparing the positional vector  $P_p$  determined by measured values and the positional vector  $C_p$  given as the computer model, and comparing the unit normal vector  $P_n$  determined by measured values and the unit normal vector  $C_n$  given as the computer model.

### CONCLUSION

This research is summarized as follows.

(1) The PN-probe based on a spherical potentiometer, which is able to directly detect the location of the contact point between the PN-probe and an object to be measured, is proposed and developed by forming a resistant thin film of TiN on a quartz glass ball.

(2) The system of measuring a positional vector and a unit normal vector of a point to be measured is constructed by combining the PN-probe with a CNC-CMM systematically.

(3) The new methods of measuring dimensions such as a distance between parallel planes and radii of a cylinder and a sphere, which are based on measurement of positional vectors and unit normal vector of arbitrary two points, are proposed.

(4) The method of evaluating an interpolating shape of a patch forming a free-curved surface based on measurement of positional vectors and unit normal vectors of sixteen arbitrary points on a patch is proposed.



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